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ANALYSIS OF THE FLOW OF RAREFIED GAS THROUGH A LAYER OF A POROUS BODY ON THE BASIS OF DIRECT NUMERICAL SOLUTION OF THE KINETIC BOLTZMANN EQUATION

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The problem of the flow of rarefied gas through a layer of a porous body, which is replaced by a homogeneous system of immobile spherical particles, is solved on the basis of the method of direct numerical solution of the kinetic Boltzmann equation. The effect of the spherical particles on the gas molecules is described as a kind of "boundary condition" distributed in space. Dependences of the density, temperature, and velocity on the coordinate in physical space are obtained; cross sections of the distribution function of the gas molecules by velocities are presented.

The present work is devoted to an investigation of the timely problem of heat and mass transfer in gas flow through a highly porous body. One of the most widespread approaches to the study of this problem is based on the use of a "dust-gas" model in which the real porous body is replaced by a homogeneous system of randomly arranged immobile spherical particles. Here, the description of the interaction of the gas and the solid particles is the principal feature. In [1–3], the expression for the gas flow is written in the form of a superposition of Knudsen and viscous flows. In this case, it is then necessary to introduce empirical coefficients.

Studies are known [4–12] where the methods of the kinetic theory are used for describing transport processes in disperse media. The paper [10], in which, on the basis of the dust-gas model, the boundary-value problem of the kinetic theory of gases is formulated and solved for mass transfer in a finite-thickness porous layer with account for the resistance of particles, i.e., the convective component of the flow, is the most similar to the problem considered in the present work. For the gas-molecule distribution function by velocities f within a porous body, we write an equation with consideration of the "external" force F, which is the result of the collective interaction of gas molecules with particles:

$$\xi_x \frac{\partial f}{\partial x} + \frac{F}{m} \frac{\partial f}{\partial \xi_x} = 0 . \tag{1}$$

Thus, the kinetic problem of gas flow in a porous medium is replaced by a one-dimensional problem, so that within the kinetic equation in place of the collision integral we introduce a term with the external force defined on the basis of the solution of the problem of flow past an individual spherical particle on condition of diffuse reflection of gas molecules from its surface.

In what follows, we give an approach to the solution of a similar problem on the basis of the Boltzmann equation which takes into account the interaction of gas with solid particles and collisions of molecules and allows one to cover a wider range of processes and phenomena:

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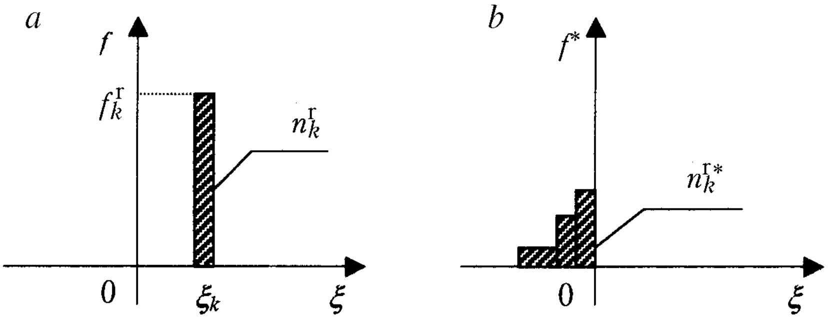

Fig. 1. Transformation of the distribution function in the interaction between gas molecules and a solid particle: a) before interaction; b) after interaction.

$$\xi_x \frac{\partial f}{\partial x} = J_{11} + J_{12}, \tag{2}$$

where J_{11} and J_{12} are the collision integrals describing the interaction between gas molecules and also the interaction of the particles of a solid body.

When the size of solid particles is rather large, collision between them can be considered as interaction with the surface using different models of gas-molecule reflection. In [11], a rather detailed review of these approaches is presented, and it is noted that this mechanism can be used now for particles consisting of several atoms. Bogdanov et al. [12] obtained a number of models of kinetic equations for a carrying gas and an admixture phase. But due to their extremely complex and cumbersome nature the corresponding numerical realization turned out to be problematic.

In the present work, we took as the basis the method of direct numerical solution of the kinetic Boltzmann equation [13] as applied to a two-component gas mixture [14]. The presence of solid particles in the mixture is considered as a kind of "boundary condition" distributed in space. In this case, the integral J_{12} is replaced by a comparatively simple computational procedure describing reflection of gas molecules from the surface of the particles. By virtue of this, here we discuss the technique of conversion of the distribution function in gas that is caused by the interaction of gas molecules with a solid body.

Within the framework of the discrete model we assume that molecules can have values of the velocities determined by a fixed velocity grid. During a finite time interval Δt, only a part of gas molecules n^r rather than all the gas molecules contained in the volume will collide with a solid particle.

Thus, the distribution function of molecules by velocities will consist of two parts: the first – the constant part f_k^n – corresponds to those molecules which do not succeed in interacting with solid particles during the time Δt, the second – f_k^r – is the part of the distribution function for molecules which will collide with a solid particle during Δt. The subscript k denotes the kth point of the velocity grid:

$$f_k = f_k^\mathrm{n} + f_k^\mathrm{r}.$$

In interaction with a solid surface, the molecules are scattered according to the adopted model, e.g., the diffuse one. The molecules, which had the same velocity ξ_k before the interaction, will acquire different velocities upon collision; these velocities occupy a certain region in the velocity space. Thus, as a result of the interaction of the molecules n_k^r with a solid particle, the part of the distribution function f_k^r is transformed, whereas f_k^n remains constant. The process of transformation of the distribution function is presented in Fig. 1.

The density of the reflected gas molecules that enters into the half-Maxwellian is determined from the nonflow condition. Similar transformations of the distribution function must be made for each velocity point.

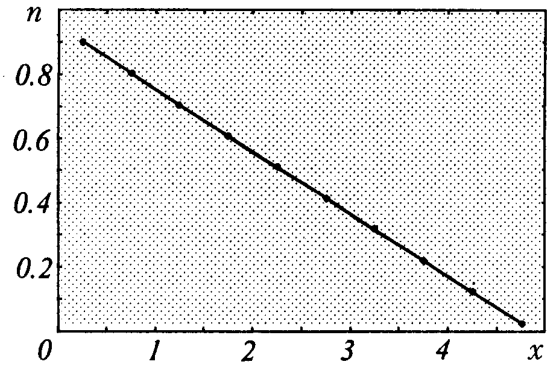

Fig. 2. Gas density n as a function of x inside a porous body.

In order to find the distribution function of gas molecules by velocities upon collision with solid particles $f_k^{\mathrm{r}*}$, one must sum up the values of the reflected functions $f^*(\xi_n)$ for all the velocity points ξ_n:

$$f_k^{\mathrm{r}*} = \sum_{n=1}^{M} f^*(\xi_n) \,.$$

Here M is the total number of velocity points, i.e., all the molecules reflected from solid particles make their own, large or small, contribution to each cell of the velocity space.

The number of gas molecules per unit volume that collided with solid particles during the time Δt is determined by the expression

$$n_k^{\mathrm{r}} = \frac{1}{4} N n_k \pi D_*^2 \, |\vec{g}| \; \Delta t \,,$$

here $D_* = D + d$, $\vec{g} = \vec{\xi_k} - \vec{v}$ is the relative velocity of motion of a gas molecule and a solid particle, $n_k = f_k \Delta\xi^3$ is the concentration of molecules having the velocity $\vec{\xi_k}$, and $\Delta\xi^3$ is the volume of the cell in the velocity space. For the part of the distribution function f_k^{r} we can write

$$f_k^{\mathrm{r}} = \frac{1}{4} N \pi D_*^2 \, |\vec{g}| \; \Delta t \,.$$

Determining $f_k^{\mathrm{r}*}$ by the above algorithm, we can find the distribution function of gas molecules by velocities upon collisions with solid particles:

$$f_k^* = f_k^{\mathrm{n}} + f_k^{\mathrm{r}*} \,.$$

Repeating this procedure for all velocity points, we can find the distribution function in the entire phase space.

On the basis of the above approach, we solved the problem of gas evaporation to a semiinfinite highly porous body. As in [10], the real porous body is modeled by a system of randomly arranged immobile spherical particles. As an example, we performed the calculation for $D = 0.5 \cdot 10^{-9}$ m and $N = 7.18 \cdot 10^{25}$ m^{-3}. The gas temperature at the inlet to the porous body was $T_0 = 300$ K and the pressure $P_0 = 20$ kPa.

Figure 2 presents the density distribution of a gas inside a porous body $n(x)$. The density and the coordinate x are given in dimensionless form: n/n_0 and x/λ_0. The dimensionless velocity is determined as $u/\sqrt{RT_0}$. The gas is nitrogen. The solid line indicates the results obtained without a collision between gas molecules, $J_{11} = 0$, and the points show the results with account for collisions, $J_{11} \neq 0$. It is seen that molecular interaction in the gas is insignificant for the given parameters of the porous body.

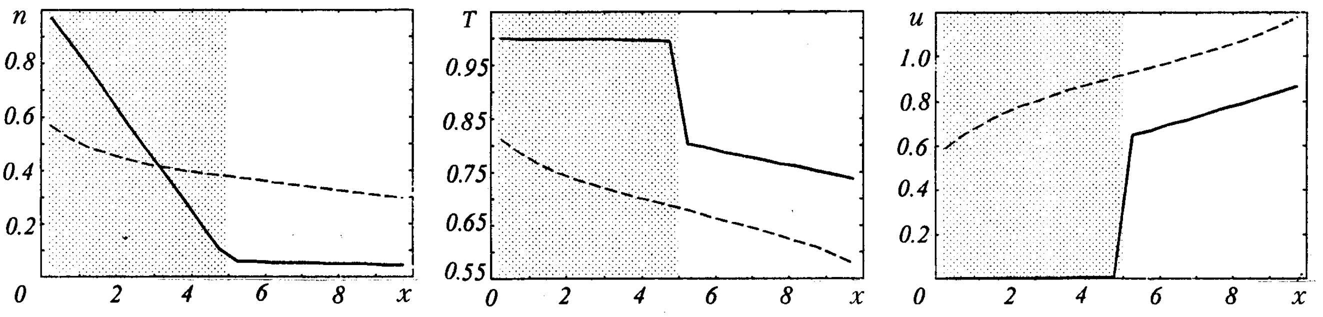

Fig. 3. Dependence of the gas density n, the temperature T, and the mean velocity of flow u on x.

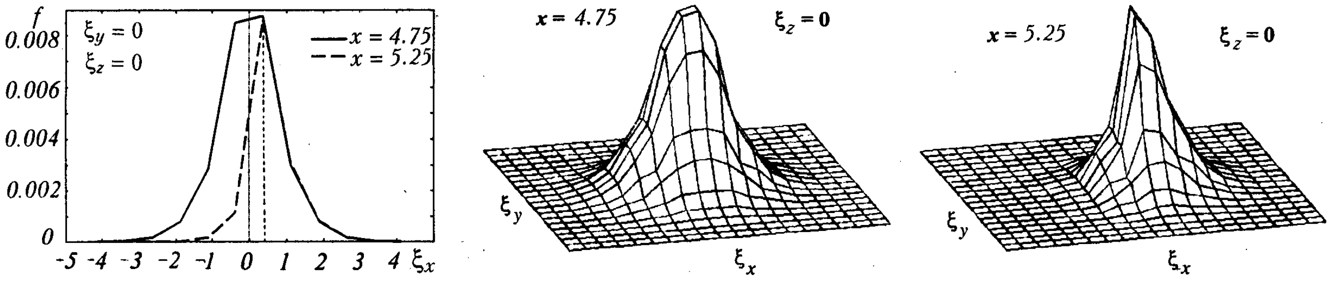

Fig. 4. Change in the distribution function of molecules by velocities upon passage from the porous body to a gas medium.

A combined problem of gas flow through a porous body was solved at the following stage. The size of the computation region along x in dimensionless units is equal to 10. The gas evaporates from the surface $x = 0$. The region $x = 0–5$ is occupied by the porous body. The surface $x = 10$ is absolutely cryogenic (the gas is absorbed completely).

Figure 3 presents the dependences of the density, temperature, and mean velocity of the gas flow on the coordinate x. The solid line shows evaporation in the presence of a porous body and the dashed line shows evaporation without it. The results obtained indicate that the porous body strongly affects the character of heat and mass transfer and upon passage from the porous body to a rarefied gas medium the macroparameters of the gas change stepwise.

Figure 4 illustrates the change in the distribution function of gas molecules by velocities at the porous body–gas interface. The cross section $x = 4.75$ lies inside the porous body near the surface (the solid line) and the cross section $x = 5.25$ lies outside (the dashed line); the dotted line shows the position of the maximum of the function. This figure also presents two-dimensional cross sections of the distribution function. It is well seen that f undergoes a substantial transformation at the interface. If, inside the porous body, the function is close to Maxwellian, outside it possesses obvious asymmetry.

The results presented in Figs. 3 and 4 indicate the substantially nonequilibrium process of passage through the interface between the porous body and the gas.

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NOTATION

f, distribution function of gas molecules by velocities; ξ_x, ξ_y, and ξ_z, components of the velocity vector $\vec{\xi}$; F, external force affecting a molecule; m, mass of the gas molecule; x, coordinate in physical space; n, concentration of the gas molecules; n^{r}, part of the total number of molecules which collide with a solid particle during a finite time interval Δt; f_k^{n}, distribution function corresponding to those molecules of the kth point of the velocity grid which do not manage to interact with solid particles during the time Δt; f_k^{r}., part of the distribution function for the molecules of the kth point of the velocity grid that will collide with a solid particle during Δt; f_k, total distribution function for the molecules of the kth point of the velocity grid; $f_k^{\mathrm{r}*}$, distribution function of gas molecules by velocities upon collision with solid particles for the kth point of the velocity grid; $f^*(\xi_n)$, distribution function of reflected gas molecules for the velocity point ξ_n; t, time; N, concentration of solid particles; D_*, sum of the diameters of the solid particle and the gas molecule; D, diameter of the solid particle; d, diameter of the gas molecule; $\vec{g}$, relative velocity of motion of the gas molecule and the solid particle; $\vec{v}$, velocity of motion of dust particles; T_0 and P_0, temperature of the gas and pressure at the inlet to the porous body; n_0, concentration of the gas molecules at T_0 and P_0; λ_0, mean-free path at T_0 and P_0; R, individual gas constant; u, mean (macroscopic) velocity of motion of the gas.

REFERENCES

1. B. V. Deryagin and S. P. Bakanov, *Zh. Tekh. Fiz.*, **27**, Issue 9, 2056–2070 (1957).
2. E. A. Mason, A. P. Malinauskas, and R. B. Evans, *J. Chem. Phys.*, **46**, 3188–3216 (1967).
3. A. A. Abramov and V. M. Zhdanov, *Teor. Osn. Khim. Tekhnol.*, **7**, No. 3, 365–372 (1973).
4. V. P. Myasnikov, *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 4, 84–89 (1967).
5. Yu. A. Buevich, *Prikl. Mat. Mekh.*, **33**, No. 3, 482–494 (1969).
6. Yu. P. Lun'kin and V. F. Mymrin, *Pis'ma Zh. Tekh. Fiz.*, **5**, Issue 3, 146–149 (1979).
7. V. A. Tsybarov, *Vestn. LGU, Mat., Mekh., Astr.*, No. 13, Issue 3, 170–176 (1976).
8. G. V. Dubrovskii, A. F. Kondratenko, and V. A. Fedotov, *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 1, 137–143 (1983).
9. Yu. N. Savchenko, in: *Molecular Gasdynamics* [in Russian], Moscow (1982), pp. 169–175.
10. V. V. Levdanskii, V. G. Leitsina, and N. V. Pavlyukevich, *Inzh.-Fiz. Zh.*, **46**, No. 6, 905–913 (1984).
11. A. V. Bogdanov, Yu. E. Gorbachev, G. V. Dubrovskii, et al., *Kinetic Theory of a Mixture of Gas and Solid Particles.* I, Preprint No. 941, Institute of Engineering Physics, USSR Academy of Sciences [in Russian], Leningrad (1985).
12. A. V. Bogdanov, Yu. E. Gorbachev, G. V. Dubrovskii, et al., *Kinetic Theory of a Mixture of Gas and Solid Particles.* II, Preprint No. 989, Institute of Engineering Physics, USSR Academy of Sciences [in Russian], Leningrad (1985).
13. V. V. Aristov and F. G. Cheremisin, *Direct Numerical Solution of the Kinetic Boltzmann Equation* [in Russian], Moscow (1992).
14. A. P. Kryukov and I. N. Shishkova, in: *Proc. 1st Russ. Nat. Conf. on Heat Transfer*, Vol. 5, *Evaporation and Condensation* [in Russian], Moscow (1994), pp. 90–96.